Statistical Ocean-Acoustics and Environmental Inversion after Stochastic Propagation and Scattering

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LONG-TERM GOAL

The long term goal of this research is to provide a unified statistical theory for ocean acoustic measurements made after stochastic propagation and scattering that includes procedures for quantitatively evaluating the performance of environmantal or target parameter estimates derived from these measurements.

OBJECTIVES

(A) In recent years, a wide variety of acoustic techniques have been developed to probe the marine environment. These techniques typically require the nonlinear inversion of acoustic field data measured on a hydrophone array. The data, however, are often randomized by the addition of naturally occurring ambient noise or by fluctuations in the waveguide's refractive index and boundaries. The nonlinear inversion of random data often leads to estimates with biases and variances that are difficult to quantify analytically. It has become popular in recent years to (1) simply assume that biases are negligible and to (2) compute limiting bounds on the variance of these nonlinear estimators, since these bounds are usually much easier to obtain than the actual variance. A major problem, however, is that the estimators may be strongly biased and that the bounds are only guaranteed to converge to the true variance of the estimator under sufficiently high signal-to-noise ratio (SNR). One of the primary objectives of this year's research is to apply higher order asymptotic theory to derive general expressions for the 1st-order-bias and 2nd-order-variance of a general maximum likelihood estimate and to apply these expressions in a variety of ocean acoustic inverse problems, such as matched field processing, matched field inversion and active sonar range estimate. The specific objectives are to (1) demonstrate that the bias in these problems can be extremely large and that the Cramer-Rao lower bound provides an unrealistically optimistic estimate of the true variance and (2) to determine requisite conditions on sample size and signal to noise ratio for the bias to become negligible and the estimators to attain minimum variance.

(B) A second primary objective of this year's research is to experimentally determine the bistatic scattering properties of bathymetric features in the deep ocean. An important part of this objective is to determine when the scattering properties can be modeled deterministically and when a stochastic model becomes necessary to explain the observed data. Another important part of this objective is to determine how bathymetric sampling will affect the estimation of scattering properties made from remote acoustic measurements.

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APPROACH

(A) If an estimator for a particular parameter, for large sample sizes or high signal to noise, is asymptotically optimal, which means that it becomes unbiased and attains the minimum variance possible, which is known as the Cramer-Rao Bound, it is guaranteed to be the maximum likelihood estimator. This fact is useful because the maximum likelihood estimate can be obtained simply by maximizing the likelihood function with respect to the parameter to be estimated. The likelihood function is simply the probability distribution function for the data, evaluated at the measured values of the data, given the parameters to be estimated which are unknown variables to be determined in the maximization. The linear least squares estimator, which for example is used extensively in ocean-acoustic inversions, is the same as the maximum likelihood estimator only when the data and parameter vectors are linearly related and the data are uncorrelated, have the same variance, follow a multivariate Gaussian distribution, and the parameters to be estimated depend only on the expected value of the data and not the covariance of the data. These assumptions are typically not satisfied in ocean-acoustic inverse problems even in large sample sizes and high signal-to-noise ratio, so the maximum likelihood approach is preferable since it will at least be asymptotically optimal. In ocean acoustics, the data and parameters to be inverted from the data typically share a complicated nonlinear relationship that often can only be described by numerically modeling using sophisticated wave propagation software. It then becomes impractical to exactly determine the variance and bias of ocean-acoustic parameter estimates analytically or even numerically since extensive computations must be made for every particular case describing parameter and data.

Our approach is to develop general analytic expressions for the bias and variance of the maximum likelihood estimate as series expansions in increasing order of the inverse sample size or inverse signal to noise ratio. The expansions are particularly useful because the number of terms necessary to describe the bias or variance decreases as the sample size or signal to noise ratio increases. The expansions are derived from a general multivariate asymptotic expansion of the maximum likelihood estimate for data following an arbitrary distribution. This expansion involves only partial derivatives of the log-likelihood function and expected values of these partial derivatives and so requires the use higher order tensor methods to describe the nonlinear and non-geometric statistical relationship between moments of the data and the parameter vectors. The order of the tensors required increases with the power of the inverse sample size and nonlinearity between data and parameter in the maximum likelihood estimate. The first order bias and first order covariance, for example, are inversely proportional to sample size. The first order covariance is the Cramer-Rao bound. The second order covariance can serve as a tighter bound than the Cramer-Rao bound, the second order term being a correction to the Cramer-Rao bound that can be used to determine when it is applicable. Analytic expressions are also derived for the sample size necessary for the first order bias to be negligible and for variance to attain the Cramer Rao bound.

The first order bias and second order covariance are then derived for the specific case of a multivariate Gaussian probability distribution along with sample size requirements for the bias to be negligible and the variance to become minimum. These expressions are then evaluated numerically for ocean-acoustic inverse problems using wave propagation and signal processing software.

(B) The bistatic scattering characteristics of two geologically distinct abyssal hills located on the western flank of the Mid-Atlantic Ridge (MAR), known as B' and C', are experimentally compared using data acquired with low-frequency towed-array systems at ½ convergence zone (~33 km) stand-off. The comparison is significant because the abyssal hills span the two classes of elevated seafloor crust that cover the Mid-Atlantic Ridge (MAR). The highly lineated B' feature is representative of abyssal hills composed of outside corner (OC) crust, the most commonly occurring category, whereas the domed C' promontory is representative of the rougher, low-aspect-ratio abyssal hills composed of inside corner (IC) crust. The latter are less common and usually restricted to segment valley margins. The mean bi-azumthal scattering distributions of the two abyssal hills are determined using high resolution bathymetry, parabolic equation modeling and raytrace. The adverse effect of using bathymetry that under-samples seafloor projected area in scattering strength analysis is also quantified with data from the B' ridge.

WORK COMPLETED

(A) By applying higher order asymptotics, general expressions are obtained for the 1st-order-bias and 2nd-order-variance of a general maximum likelihood estimate [1,2]. These expressions are applied in a variety of ocean-acoustic inverse problems including the matched field localization of a source in a shallow-water waveguide [1] and the range-estimation and Doppler shift-estimation problem of active sonar [2].

In the matched field processing application we examined the matched field localization of a source in a 100-m deep Pekeris waveguide radiating at 100 Hz over a sand bottom in Kuperman-Ingenito waveguide noise with a 10-element vertical array at $\lambda/2$ –spacing. The signal-to-noise ratio (SNR) is defined as 10log(sum of signal intensity across array/sum of the noise intensity across the array) so that the signal to noise ratio decreases as the source moves from the array. Using the coherent array gain of up to 10 dB possible for the given array, matched field processing for the range-depth localization of the source is possible for SNRs approaching –10 dB. We find, however, that for SNRs between 0 and –10 dB the first order bias of the matched field processor is on the order of 20 m, a significant portion of the waveguide depth. Only as the SNR increases above 0 dB does the depth bias become negligible. We find that averaging on the order of ten independent time or frequency samples is necessary to reduce the depth bias to tolerable values. We find that the range bias is never a significant fraction of the range to the target in the operation SNR range of up to –10 dB SNR range. The second order variance term of the maximum likelihood estimate, however, is greater than or on the order of the Cramer-Rao bound, the first order term, for SNRs less than roughly 10 dB for both range and depth estimation. The Cramer-Rao bound is therefore a very poor estimate of the variance of range and depth estimates by matched field processing except in extremely high SNR, at least roughly 20 dB higher than the operational limit set by array gain. Hundreds to thousands of independent samples are needed in a time or frequency average to reduce the variance to the Cramer-Rao bound in the 0 to –10 dB SNR range.

We have applied our asymptotic expressions to determine analytic conditions in which the matched filter attains the Cramer-Rao bound in sonar range estimation and Doppler shift estimation problems. We examined three common signal waveforms, the linear frequency modulated (LFM) pulse, the hyperbolic frequency modulated (HFM) pulse, and the Gaussian pulse. We find that in all cases the SNR must be in excess of 20 dB before the variance of the range or Doppler shift estimate can attain the Cramer-Rao bound. We also find that while the first order range resolution increases with increasing signal bandwidth so does the second order variance in relation to the first order variance.

This occurs because the increase in bandwidth introduces greater nonlinearity in the signal which imposes a higher SNR requirement for the Cramer-Rao bound to me met.

(B) The bistatic scattering characteristics of two geologically distinct abyssal hills located on the western flank of the Mid-Atlantic Ridge (MAR), composed of *outside corner* and *inside corner* crust and referred to as B' and C' respectively, are experimentally compared [3,4]. The levels of bistatic reverberation, measured from scarps on the two abyssal hills in bistatic experiments from ½ CZ stand-off, exhibit nearly identical, constant azimuthal dependencies. The mean bi-azimuthal scattering distributions of scarps on the two abyssal hills are also found to exhibit nearly identical and constant azimuthal dependencies with mean strength equal to –11 dB *when estimated from supporting bathymetry sampled at* 200-m *intervals*. Higher resolution supporting bathymetry, only available at B' and not at C', sampled at 5-m intervals reveals that the projected area of the B' scarps, as seen by refracted rays traveling from source to bistatic receiver at ½ CZ, is significantly under-sampled with the 200-m sampled bathymetry. This under-sampling leads to a uniform bias of roughly –6 dB in the level of modeled bistatic reverbation from the B' scarps and, consequently, a uniform bias of +6 dB in the strength of the mean bi-azimuthal scattering distributions of the B' scarps. The strength of the mean bi-azimuthal scattering distributions of the B' scarps is more accurately given by the constant –17 dB +/– 8dB when estimated from the high-resolution bathymetry sampled at 5-m intervals. A general conclusion is that the use of bathymetry that under-samples the projected area of the seafloor within the resolution footprint of the towed-array system can lead to significant over-estimates in the strength of seafloor scattering.

RESULTS

(A) We have derived general expressions for the first order bias and second order covariance of a general multivariate maximum likelihood estimate. The second order covariance is a tighter bound than the Cramer-Rao lower bound, the most widely used bound in statistics, but requires more effort to implement. We have applied these results to derive analytic expressions for the first order bias and second order variance of a multivariate maximum likelihood estimate from Gaussian data. We have shown that when matched field processing is used to estimate the range and depth of a source in an ocean waveguide significant biases in depth estimation can occur and that the Cramer-Rao bound provides an unrealistically optimistic estimate of the true range and depth variances unless the signal to noise ratio is very high.

We have applied our asymptotic expressions to determine analytic conditions in which the matched filter attains the Cramer-Rao bound in sonar range estimation and Doppler shift estimation problems. We find that the SNR must typically be in excess of 20 dB before the variance of the range or Doppler shift estimate can attain the Cramer-Rao bound. We also find that while the first order range resolution increases with increasing signal bandwidth so does the second order variance in relation to the first order variance.

We have developed rigorous methods to show that the Cramer-Rao bound provides an unrealistically optimistic estimate of the true variance and the bias can be significant in many practical ocean acoustic estimation problems. We have developed rigorous methods to determine when the bias is negligible and when the Cramer-Rao bound can be attained.

(B) We find that long-range reverberation from prominent geological features of the Mid-Atlantic Ridge, and likely other mid-ocean ridges, can be adequately modeled as having Lambertian scattering characteristics. We hypothesize that the albedo of $\pi/10^{1.7}$, measured for the two major scarps on the B' abyssal hill with a more than adequate bathymetric sampling density, provides a reasonable estimate of the albedo of all abyssal hills comprised of *outside corner* crust and may also provide a good estimate of the albedo of abyssal hills comprised of *inside corner* crust. We take the albedo of $\pi/10^{1.1}$, measured for an abyssal hill comprised of *inside corner* crust from potentially undersampled bathymetry, as an upper bound on albedos of abyssal hills comprised of *inside corner* crust.

IMPACT/APPLICATION

(A) Our expressions for the first order bias and second order covariance of the maximum likelihood estimate are of a fundamental and completely general nature so that they can be used in any statisical estimation problem. Our next ocean-acoustic application will be in matched field inversion of oceanographic and geophysical paramters.

(B) We now have an empirical model for the bistatic scattering distribution of deep ocean abyssal hills and have developed requirements on bathymetric sampling necessary to extract the scattering properties of geologic features from acoustic measurements. The former results are applicable for sonar operations in deep water the latter are applicable in the design of experiments to determine the scattering properties of geologic features.

TRANSITIONS

(A) The statistical tools developed in this program are currently being applied in every other program the PI is engaged in, including the SECNAV/CNO Scholar work on ocean-acoustic signal processing, the ONR Geoclutter program, and the ONR Underwater Signal processing fluctuating target program.

(B) The methods developed for bistatic scattering analysis from geologic features in deep water are presently being modified for application in the analysis of bistatic scattering from geologic features in shallow water in the ONR Geoclutter program.

RELATED PROJECTS

Same as Transitions.

PUBLICATIONS AND REFERNCES

[1] N. C. Makris and E. Naftali, “Conditions for ocean-acoustic matched field inversions to be unbiased and attain the Cramer-Rao bound ,” J. Acoust. Soc. Am. 106, 1999.

[2] E. Naftali, “First order bias and second order covariance of the multivariate Gaussian maximum likelihood estimator with application to time-delay and Doppler shift estimation,” MIT Master’s Thesis under Prof. N.C. Makris, 1999.

[3] C. S. Chia, L. Fialkowski and N. C. Makris, "A comparison of bistatic scattering from two geologically distinct abyssal hills," submitted to J. Acoust. Soc. Am.

[4] C. S. Chia, "A comparison of bistatic scattering from two geologically distinct abyssal hills," MIT Master's Thesis under Prof. N.C. Makris, 1999.

[5] N.C. Makris, C. S. Chia and L. Fialkowski, "The bi-azimuthal scattering distribution function of an abyssal hill," J. Acoust. Soc. Am in press for Nov. 1999.

[6] G. Xu, "Maximum likelihood time domain beamforming with simulated annealing," MIT Master's Thesis under Prof. N.C. Makris, 1999.